

Lesson 32: The Fundamental Theorem of Calculus (2)

Recall: $\int_a^b f(x) dx = F(b) - F(a)$ where $F'(x) = f(x)$.

So $F(b) - F(a) = \int_a^b F'(x) dx$.

Ex 1 If $\frac{dh}{dt} = 1.4t + 7$ models the growth rate of a plant, find how much the plant grows from $t=0$ to $t=5$.

$$\begin{aligned} \text{we want } h(5) - h(0) &= \int_0^5 h'(t) dt \stackrel{= \frac{dh}{dt}}{=} \\ &= \int_0^5 1.4t + 7 dt \\ &= 1.4 \left(\frac{1}{2} t^2 \right) + 7t \Big|_0^5 \\ &= .7(5)^2 + 7(5) - 0 \\ &= \boxed{52.5} \end{aligned}$$

Ex 2 Displacement is the change in position.
If $v(t) = 2e^t$, find the displacement from $t=0$ to $t=5$.
 $s'(t)$ $s(5) - s(0)$

$$\begin{aligned} s(5) - s(0) &= \int_0^5 2e^t dt \\ &= 2e^t \Big|_0^5 = \boxed{2e^5 - 2} \end{aligned}$$

When is the displacement 1?

$$\begin{aligned} \underbrace{s(x) - s(0)}_{\substack{\text{displacement} \\ \text{at } t=x}} &= \int_0^x 2e^t dt = 2e^t \Big|_0^x = 2e^x - 2 \stackrel{= 1}{=} \\ &2e^x = 3 \\ &e^x = 3/2 \end{aligned}$$

$$\boxed{x = \ln(3/2)}$$